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### Semester One Examination, 2018

### Question/Answer booklet

# MATHEMATICS

**SOLUTIONS**

**SPECIALIST**

**UNIT 1**

## Section Two:

## Calculator-assumed

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working  time (minutes) | Marks available | Percentage of examination |
| Section One:  Calculator-free | 8 | 8 | 50 | 53 | 35 |
| Section Two:  Calculator-assumed | 13 | 13 | 100 | 97 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question  
have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

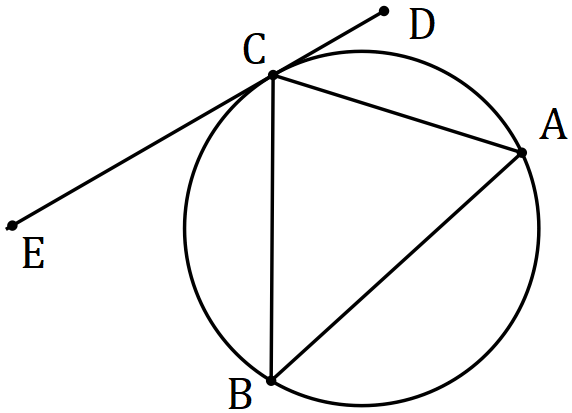
Section Two: Calculator-assumed 65% (97 Marks)

This section has**thirteen (****13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 (7 marks)

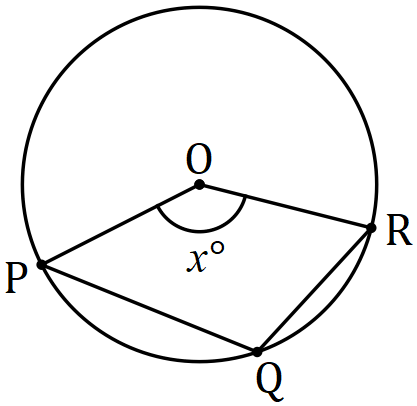
(a) In the diagram below, points and lie on a circle, is a tangent to the circle at , and . Determine the sizes of and . (3 marks)



|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓  ✓  ✓ |

(b) In the next diagram, and lie on a circle with centre and . Determine, with reasons, the size of in terms of . (4 marks)

|  |
| --- |
| **Solution** |
| (Angle sum at point)    (Angle on arc at centre twice that on circumference) |
| **Specific behaviours** |
| ✓ expression for reflex angle  ✓ reason  ✓  ✓ reason  *(Or other methods)* |



Question 10 (7 marks)

(a) A body travels with a velocity ms-1. Determine its speed and the bearing on which it is moving, assuming the positive -axis to be due north. (3 marks)

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| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ speed  ✓ angle  ✓ bearing |

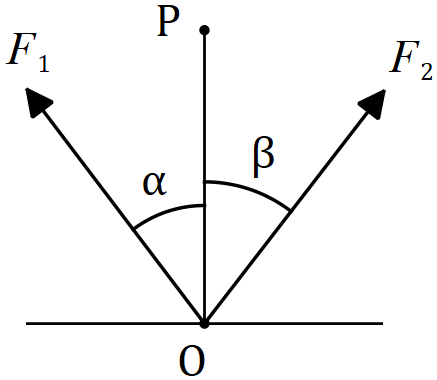
(b) Given that , determine the values of and .

(4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ equates -coefficients  ✓ equates -coefficients  ✓ value of  ✓ value of |

Question 11 (8 marks)

Two forces, and , act on a body at , and make angles of and respectively with the vertical , as shown in the diagram below.



(a) Determine the magnitude of the resultant force and the angle it makes with the vertical.

(5 marks)

|  |  |
| --- | --- |
| **Solution** | |
|  |  |
| **Specific behaviours** | |
| ✓ sketch with forces nose to tail  ✓ indicates use of cosine rule for magnitude  ✓ magnitude  ✓ indicates use of sine rule for angle  ✓ angle with vertical | |

(b) The magnitude of is to be adjusted so that the direction of the resultant is vertical. Determine the required magnitude of . (3 marks)

|  |  |
| --- | --- |
| **Solution** | |
|  |  |
| **Specific behaviours** | |
| ✓ sketch  ✓ indicates use of sine rule  ✓ magnitude | |

Question 12 (6 marks)

The largest Australian family recently met with the largest English family. Between them, these two families had children.

(a) Three of the children were chosen at random to feature in a TV documentary about the two families. Determine the number of different selections of three children that were possible. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct number |

(b) Prove that at least four of the children were born in the same month of the year.

(2 marks)

|  |
| --- |
| **Solution** |
| Let the 12 months of the year be the pigeon-holes and the 37 children the pigeons. If 3 pigeons are placed in each of the 12 pigeon-holes, then there is still one left over, and so at least one of the pigeon-holes must have at least 4 pigeons (children). |
| **Specific behaviours** |
| ✓ defines pigeons and pigeon-holes  ✓ uses pigeon-hole principle |

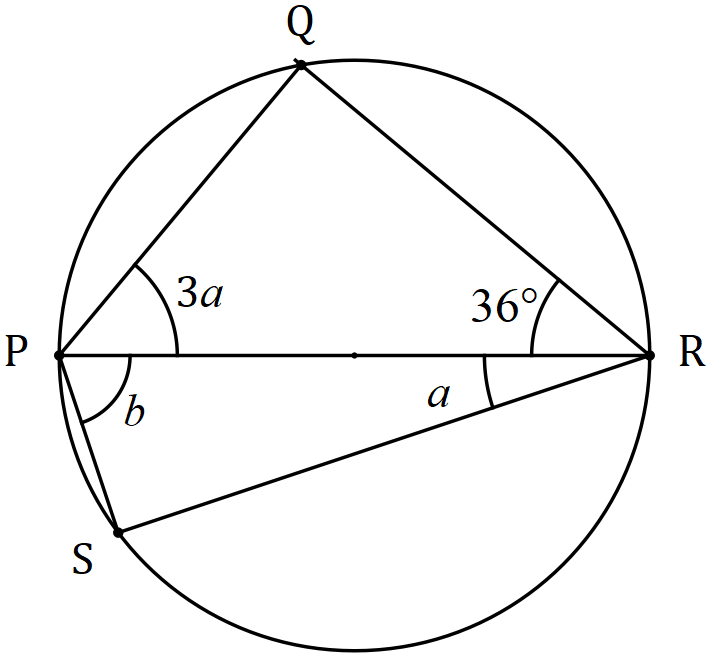
There were more children in the English family than the Australian family and the English children all had blue, brown, hazel or grey coloured eyes.

(c) Show that at least five English children had the same eye colour. (3 marks)

|  |
| --- |
| **Solution** |
| eye colours are the pigeon-holes and children are pigeons. If pigeons placed in each pigeon hole there are left over and so at least one of the pigeon-holes must have at least pigeons (children with same coloured eyes). |
| **Specific behaviours** |
| ✓ minimum number of English children  ✓ defines pigeons and pigeon-holes  ✓ uses pigeon-hole principle |

Question 13 (8 marks)

(a) Determine the size of angles and in the diagram below, where and lie on the circumference of the circle with diameter . (3 marks)



|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses angle in semi-circle  ✓ value of  ✓ value of |

(b) Triangle has sides of length cm, cm and cm. Prove, using the method of contradiction, that if is a diameter of a circle then does not lie on the circumference of the circle. (5 marks)

|  |
| --- |
| **Solution** |
| Assume that does lie on the circumference of the circle.  Then (angle in semi-circle)  Hence (Pythagoras' theorem)  But .  Hence assumption must be incorrect and so does not lie on the circumference. |
| **Specific behaviours** |
| ✓ assumes contradictory statement  ✓ states angle in semi-circle  ✓ deduces relationship between sides  ✓ shows contradiction  ✓ summarises |

Question 14 (8 marks)

(a) Simplify given that and vector is parallel and in the opposite direction to vector . (4 marks)

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| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expands scalar product  ✓ indicates  ✓ substitutes magnitudes  ✓ simplifies |

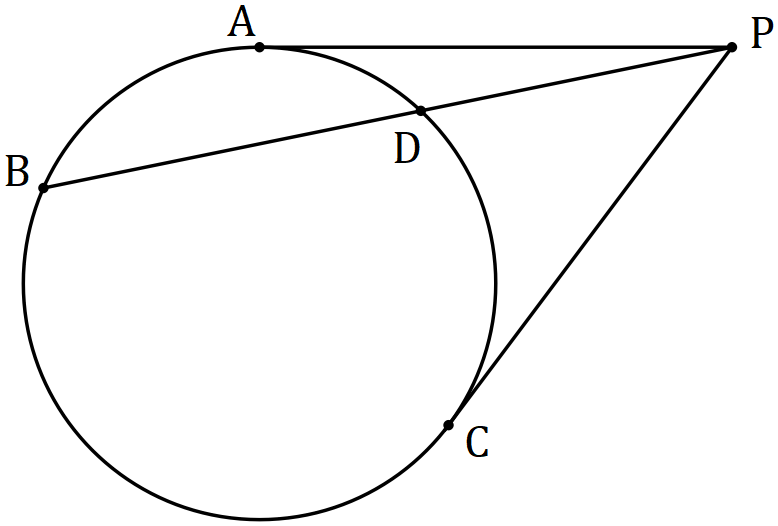
(b) Using and , demonstrate a vector method to show that if the diagonals and of parallelogram are perpendicular, then the parallelogram is a rhombus.

(4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ determines vectors for diagonals  ✓ uses scalar product  ✓ expands scalar product  ✓ explains that sides must be congruent |

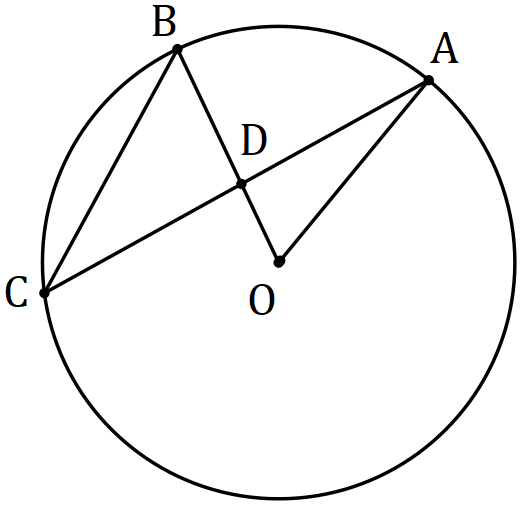
Question 15 (9 marks)

(a) In the diagram below, and are tangents to the circle, with cm. Secant cuts the circle at , so that cm. Determine the lengths of and . (4 marks)



|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ value of  ✓ indicates use of tangent-secant theorem  ✓ equation for  ✓ value of |

(b) In the diagram below, and lie on the circumference of the circle with centre , with intersecting at . Prove that . (5 marks)



|  |
| --- |
| **Solution** |
| (sum of exterior angles equal)  But  (angle at centre-circumference)  Hence  And so |
| **Specific behaviours** |
| ✓ derives first equation  ✓ reasoning for first equation  ✓ uses angle at centre-circumference  ✓ substitutes  ✓ simplifies |

Question 16 (9 marks)

(a) Determine the number of integers between and that are divisible by or .

(4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ divisible by 4 & 7  ✓ divisible by 28  ✓ use of inclusion-exclusion principle  ✓ correct number |

(b) A pigeon fancier has Florentine, King and Maltese pigeons and must choose three of them to enter in a local show. Determine the number of different ways the three pigeons can be chosen if

(i) there are no restrictions. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct number |

(ii) the fancier decides to take one of each breed. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses multiplication principle  ✓ correct number |

(iii) the fancier decides to take at least two Maltese pigeons. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates two cases  ✓ correct number |

Question 17 (6 marks)

Three vectors are and .

(a) Determine the vector projection of on in exact form. (2 marks)

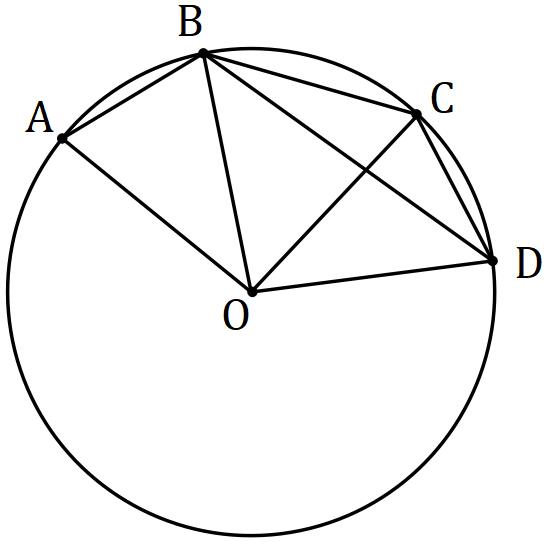
|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates suitable form of projection  ✓ solution in exact form |

(b) If has the same magnitude as and is perpendicular to , determine the exact values of the coefficients and . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ equation from magnitudes  ✓ equation from perpendicular  ✓ one solution  ✓ both solutions |

Question 18 (7 marks)

(a) In the diagram below, points and lie on the minor arc of the circle with centre . The lengths of chords and are congruent, and . Determine the size of . (3 marks)



|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates equal angles on equal chords  ✓ size of  ✓ size of |

(b) Line segment intersects line segment at . Given that and are non-parallel and the lengths and are 6, 41, 21 and 31 cm respectively, explain whether the points and are concyclic. (4 marks)

|  |
| --- |
| **Solution** |
| Concyclic, as interval lengths satisfy the intersecting chord theorem. |
| **Specific behaviours** |
| ✓ sketch  ✓ uses correct chord lengths  ✓ uses property of intersecting chords  ✓ explanation |

Question 19 (8 marks)

(a) Triangle has vertices with position vectors and . Point lies on side so that . Determine the vector . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓  ✓  ✓  ✓ |

(b) is a parallelogram. Point is the midpoint of side and point is on side so that . If and , determine in terms of and . (4 marks)

|  |  |
| --- | --- |
| **Solution** | |
|  |  |
| **Specific behaviours** | |
| ✓ sketch  ✓  ✓  ✓ | |

Question 20 (7 marks)

A small boat leaves jetty to travel to jetty , m away on a bearing of . A steady current of ms-1 runs in the river between the jetties on a bearing . If the small boat travels at a constant speed of ms-1, determine the bearing it should steer to reach jetty and how long the journey will take.

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ diagram  ✓ angle in triangle between current and  ✓ equation using sin rule for  ✓ solves for angle offset  ✓ equation using sin rule for  ✓ correct time  ✓ correct bearing |

Question 21 (7 marks)

A child is playing with thirteen coloured cubes, all the same size. There are six pink cubes, three navy and one each of red, blue, orange and green.

(a) If the child stacks cubes one on top of another to make a column, determine the number of different coloured columns that can be made using

(i) all the red, blue and green cubes. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ number |

(ii) all the pink, red and orange cubes. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ numerator  ✓ correct number |

(iii) all the cubes. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expression  ✓ correct number |

(b) If all but one of the cubes are used to make a column, determine the number of different coloured columns that can now be made. Justify your answer. (2 marks)

|  |
| --- |
| **Solution** |
| All the columns 13 tall with a pink on top must have a difference in the 12 cubes beneath and so if the top pink is removed, the remaining columns will still be different.  The same is true for columns with other coloured top cubes, and the remaining 12 tall columns will have one less cube of the top colour and so must be different to all other columns.  So, no change. |
| **Specific behaviours** |
| ✓ correct number  ✓ justification |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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